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LETTER TO THE EDITOR

Dear Sir,
***Frictional slip between a layer and a substrate due to a periodic surface force by M. Comninou and J. R. Barber, Int. J. Solids Structures* 19(6), 533–539 (1983).**

This interesting paper raises the question of shakedown in frictional systems. In the theory of plasticity Melan's Theorem [1] states that if a system of self equilibrating residual stresses can be postulated which, together with the stresses due to the cycle of imposed loads, do not violate the yield criterion then, after repeated loading cycles, the system will shakedown to a purely elastic cycle of stress. There has been some speculation [2] whether this theorem can be applied to problems of frictional slip, such as the present one, where residual stresses are introduced by previous cycles of load. The proof of Melan's Theorem [3] relies upon the "normality" of the plastic strain increment vector to the yield surface in a stable plastic material. This condition does not apply to simple slipping friction, where the direction of slip is opposed to the frictional stress rather than the resultant stress. To my knowledge the equivalent of Melan's Theorem in slipping systems has not been proved. Nevertheless it is of interest to put it to an empirical test in the present case. First we neglect residual *normal* stresses. Figure 8 suggests that in the regions of incipient slip, $x = \pm a$, they are small, of order 0.01 p_0. The accompying figure shows $S(x)$ and $-\mu N(x)$ due to the applied load Q, drawn for the load for which $S(x)_{max}$ ($= Q/2\pi a$ at $x = \pm a$) is just equal to μp_0, if slip were in some way prevented. Now slip would be prevented if a residual stress existed of magnitude equal to the shaded area in the figure since

$$|S(x) + S_r(x)| \leqq \mu N$$

throughout the interface. Note that these residual stresses are equal but of opposite sign on each side of the centre line implying opposite directions of slip.

A system of residual stresses has then been proposed which together with the stresses due to the cyclic application of Q result in no slip. If Melan's Theorem were true for frictional slip, we could then claim that the system would shakedown under this load, even though the actual residual stress differs from those postulated in the figure. Furthermore the figure is drawn for the maximum value of $\pm Q$ for shakedown, since any increase in $S(x)$ would cause reverse slip when Q was reversed. Thus the shakedown limit is given by

$$S(x)_{max} = \mu p_0,$$

i.e.

$$Q/2\pi a = \mu p_0$$

or

$$\lambda \equiv \frac{Q}{a p_0} = 2\,\mu\pi = 3.14 \text{ when } \mu = 0.5.$$

This value compares well with the result of the paper but has, of course, been obtained without the elaborate calculations involved in following this history of slip. The maximum value of $S_r(x)$ for shakedown is 0.273 p_0 at $x = 0.826\,a$ which again compares well with the result from Fig. 7 of the paper.

The results of this paper strongly suggest that Melan's Theorem can be applied to the present problem. They present a challenge to the theoreticians to establish a shakedown theorem for frictional systems and to specify the condition in which it might apply.

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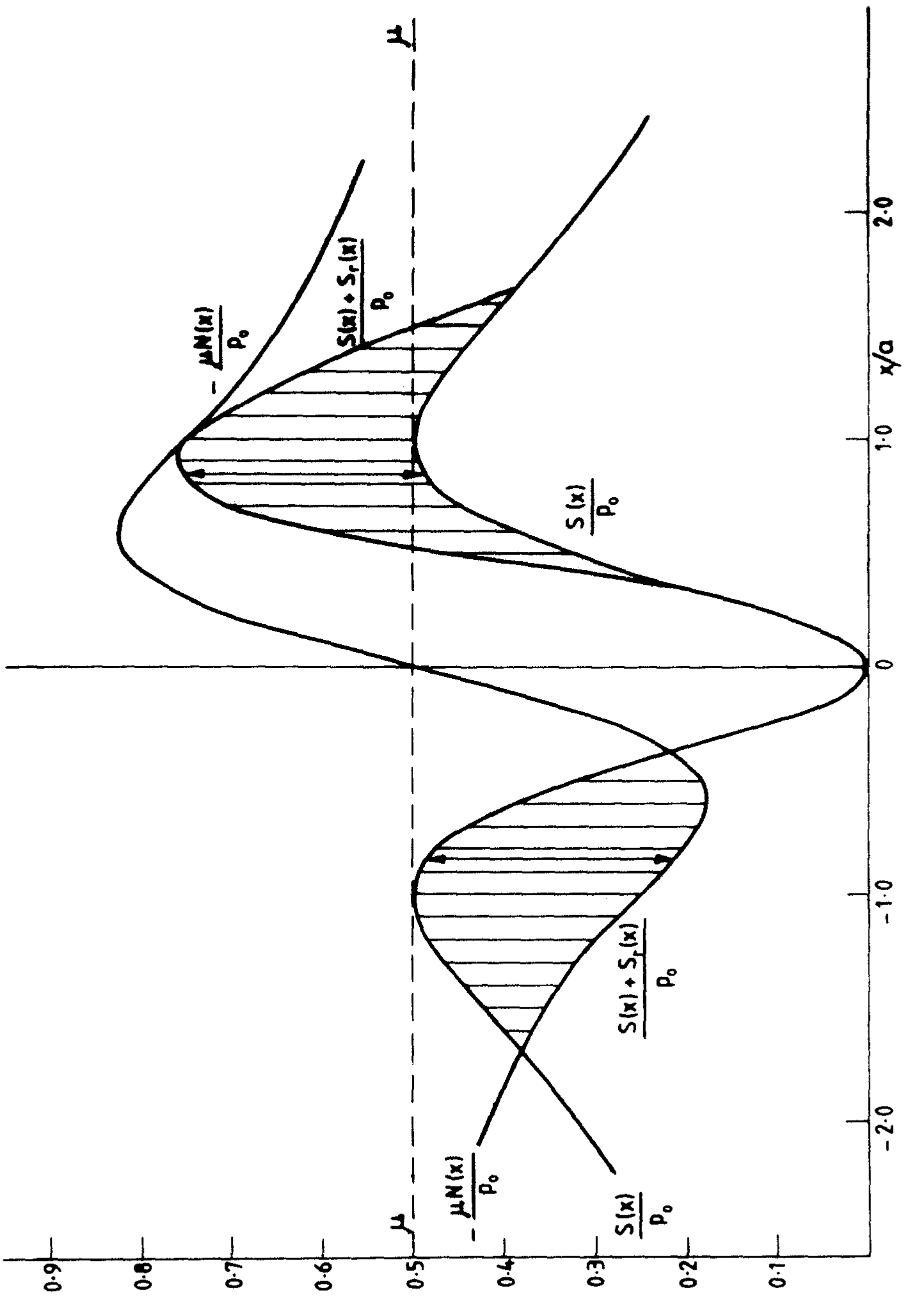

Fig. 1.